

3. Exercise Discrete Geometrie II

Deadline: 05.11.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (the flip graph)

A subdivision T of a point configuration is *almost a triangulation* if it is not a triangulation but all its proper refinements are triangulations. Two triangulations of the same point configuration are connected via a flip supported on the almost triangulation T if they are the only two triangulations refining T . Describe all triangulations and the flip-graph of the prism over a triangle.

2. Exercise (planar convex hull complexity)

Consider a convex hull algorithm that takes points v_1, v_2, \dots, v_m in \mathbb{R}^2 and returns the vertices of $\text{conv}v_1, v_2, \dots, v_m$ in cyclic order. Reduce the problem of sorting (distinct) real numbers $b_1, b_2, \dots, b_m \in \mathbb{R}$ to the above algorithm, i.e., find an efficient procedure to sort the numbers that uses the convex hull algorithm. What is the run time of your procedure and what is the run time of a sorting algorithm under the assumption that the algorithm uses pairwise comparison? What does this reduction imply for the complexity for convex hull algorithms?

3. Exercise (tropical matrices)

Consider the 2×2 tropical matrices $A = \begin{pmatrix} 2 & 3 \\ 5 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix}$. Compute the tropical product $A \odot B$ and $A \oplus B$ tropically. Also compute

$$A \oplus A^2 \oplus A^3 \oplus \dots \oplus A^{1000}.$$

4. Exercise (tropical linear system)

Let $A \in \mathbb{R}^{2 \times 2}$ be a square matrix with two rows and columns. For which vectors $b \in \mathbb{R}^2$ does the tropical linear system $A \odot x = b$ have a solution?