Technical University Berlin **WS 2025**
Faculty II, Institute for Mathematics
Secretariat MA 6–2, Erik Oor
Prof. Dr. Michael Joswig
Marcel Wack.

1. Exercise Discrete Geometrie II

Review Discrete Geometrie I

Deadline: 22.10.2025 (before the Exercise class)

Each answer should be sufficiently proven.

1. Exercise (f-vectors of polytopes)

(i) The product of two polytops $P \subset R^d$ and $Q \subset R^e$ is
$$P \times Q = \{(v, w) \in \mathbb{R}^{d+e} \mid v \in P \cup w \in Q\}.$$
Give a formula for the f-vector of $P \times Q$.

(ii) Let $P \subset P^d$ be a polytope with 0 in the relative interior. The bipyramid B over P is the convex hull
$$B = \operatorname{conv}\left(\{(v, 0) \in R^{d+1} \mid v \in P\} \cup\right)$$

2. Exercise (Linear program, basics)

Give solutions to the following linear programs.

(i) Let $c \in \mathbb{R}^n$,
$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & c^T x \\ \text{subject to} \quad & -1 \leq \mathbf{1}^T x \leq 1 \end{aligned}$$

(ii) Let $c \in \mathbb{R}^n$, $k \in \{1, \ldots n\}$,
$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & c^T x \\ \text{subject to} \quad & \mathbf{1}^T x \leq k \\ & 0 \leq \mathbf{1}^T x \leq 1 \end{aligned}$$

3. Exercise (Convex hull)

Prove or disprove: Given two sets $\mathcal{X}, \mathcal{Y} \in \mathbb{R}^n$, the intersection of the convex hull of X and Y is itself convex and
$$\operatorname{conv}(\mathcal{X}) \cap \operatorname{conv}(\mathcal{Y}) = \operatorname{conv}(\operatorname{conv}(\mathcal{X}) \cap \operatorname{conv}(\mathcal{Y}))$$

4. Exercise (Linear programming, examples)

Derive the dual program for the following linear programs of the form

$$\begin{aligned} \underset{x\in\mathbb{R}^2}{\text{maximize}} \quad & c^T x \\ \text{subject to} \quad & Ax \le b \\ & x \ge 0 \end{aligned}$$

and calculate a solution using a graphical approach.

(i) $c = \big(2, 6)\big)^T$, $A = \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix}$, $b = \big(9, 6\big)^T$

(ii) $c = \big(4, 6\big)^T$, $A = \begin{pmatrix} 10 & 12 \\ 2 & 6 \end{pmatrix}$, $b = \big(2, 8\big)^T$

5. Exercise (Linear programming)

Consider the Maximum bipartite matching problem: Given a finite bipartite graph $G = (A\dot{\cup}B, E)$, find a set of edges $S \subseteq E$ of edges such that no pair of edges in S share a common node and the cardinality $\#S$ is maximal.

(i) Model this as an integer linear program.

(ii) Dualize the LP-relaxation of that problem.

(iii) What does the optimal solution of the dual problem describe in terms of G.