ALGORITHMIC DISCRETE MATHEMATICS III: EXERCISES 5

MICHAEL JOSWIG

Exercise 1. Give an explicit algorithmic formulation of the shadow vertex algorithm.

Exercise 2. Consider the affine halfspace

$$H_n^+ \;=\; \left\{ x \in \mathbb{R}^n \;\middle|\; \sum x_i \le 3/2 \right\} \ .$$

The *dwarfed n-cube* is the polytope

$$D_n \;=\; [0,1]^n \cap H_n^+ \ .$$

a. Show that D_n is simple.
b. How many vertices and facets does D_n have?
c. Show that each simple n-polytope P with $f_{n-1}(P) = f_{n-1}(D_n)$ we have

$$f_0(P) \;\ge\; f_0(D_n) \ ,$$

i.e., D_n has the minimal number of vertices among the simple n-polytopes with the same number of facets.

Exercise 3. Describe a fast algorithm for computing the f-vector of a simple polytope. What is its complexity?

Let τ be a polygonal curve in $\mathbb{R}^n$ parameterized over an interval $[a,b]$. Then the *total curvature* $\kappa(\tau,[a,b])$ is defined as the sum of the angles between the consecutive segments of the curve.

Exercise 4. Show that the total curvature of a closed polygonal curve in the plane is an integer multiple of 2π.

Let σ be a twice continuously differentiable curve in $\mathbb{R}^n$ parameterized over the interval $[a,b]$ by arc length. Then the *total curvature* $\kappa(\sigma,[a,b])$ is defined as $\int_a^b \|\sigma''(t)\|\,dt$.

Exercise 5.

a. What is the total curvature of the unit circle? How does the curvature of an arbitrary circle depend on the radius?
b. Compute the total curvature of the plane curve $(t, \sin(t))$ in the interval $[0, 2\pi]$. Do not forget to check the parametrization!
c. How are the two notions of total curvature, for polygonal and smooth curves, related?

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