

**ALGORITHMIC DISCRETE MATHEMATICS III:
EXERCISES 4**

MICHAEL JOSWIG

Exercise 1. We consider the pivot rule RANDOM-FACET.

- a. Determine the expected path lengths $F_3(x)$ for all vertices x of the 3-dimensional Klee-Minty cube $\text{KM}(3, \epsilon)$.
- b. What is the expected path length F_n for a random vertex of $\text{KM}(n, \epsilon)$ for arbitrary $n \geq 1$?

Exercise 2. For $k \geq 3$ let

$$P_k = \text{conv} \left\{ \left(\cos \frac{2\pi i}{k}, \sin \frac{2\pi i}{k} \right) \mid 0 \leq i < k \right\}$$

be a regular k -gon in \mathbb{R}^2 . We are concerned with products $P_k \times P_\ell$ in \mathbb{R}^4 .

- a. For which k and ℓ is the linear objective function $\sum x_i$ nondegenerate on $P_k \times P_\ell$?
- b. Analyze RANDOM-EDGE on $\text{LP}(P_k \times P_\ell, \sum x_i)$.
- c. Analyze RANDOM-FACET on $\text{LP}(P_k \times P_\ell, \sum x_i)$.

Exercise 3. Devise other pivoting strategies. What would be particularly good input for them, and what would be particularly bad input?

Exercise 4. Invent and discuss various models for random polytopes. What about random linear programs?