ALGORITHMIC DISCRETE MATHEMATICS III: EXERCISES 1

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The convex hull of finitely many points with 0/1-coordinates is a *0/1-polytope*.

Exercise 1. Give bounds for the vertex and facet complexities of the 0/1-polytopes in dimension n.

Two polytopes are *combinatorially isomorphic* if their face posets are isomorphic (as posets).

Exercise 2. Enumerate the 3-dimensional 0/1-polytopes up to combinatorial automorphisms. Can you make `polymake` visualize them all in one picture?

Exercise 3. Let P and Q be convex polytope. Describe an algorithm for deciding whether P and Q are combinatorially isomorphic. Do some examples with `polymake`.

A *Schlegel diagram* of a polytope is a central projection onto one of its facets, say F, yielding a polyhedral subdivision of F by coning with the center of the projection over all facets other than F (and intersecting with F).

Exercise 4. Construct a 4-dimensional polytope which is not combinatorially isomorphic to any 0/1-polytope. Let `polymake` draw a Schlegel diagram. Generalize this to arbitrary dimension.

A polyhedron is *pointed* if it does not contain any affine subspace of positive dimension.

Exercise 5. Let $P \subset \mathbb{R}^n$ be a polyhedron. Show that P is pointed if and only if there is an affine transformation T such that $T \cdot P$ is contained in the nonnegative orthant $\mathbb{R}^n_{\geq 0}$. What can you say about projective transformations?

Exercise 6 (Recap)**.** Phase II of the simplex method for linear programming requires to start with one known vertex of the feasible region. Describe a method for phase I, i.e., an algorithm which takes $A \in \mathbb{Q}^{m\times n}$ and $b \in \mathbb{Q}^m$ as input and produces one vertex of $P(A, b) = \{x \in \mathbb{Q}^m \mid Ax \leq b\}$. You may assume that $P(A, b)$ is pointed.

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